



## Path tracking for Drones

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# Introduction

Indoors



Outdoors

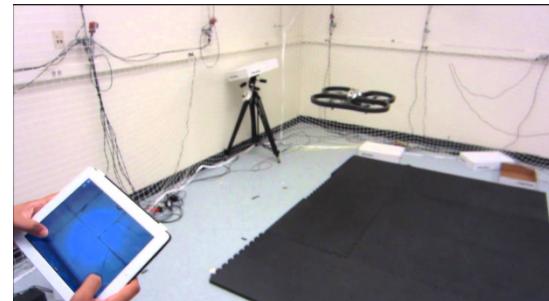


# Introduction

## Test indoors

### Pros

- Controlled environment
- Accuracy in the measure
- No GPS need it



### Cons

- Limit space for navigate
- Need a camera for navigate without a motion capture system
- There are not applications

## Test outdoors

### Pros

- More challenge to develop control laws
- More space to navigate
- A lot of applications



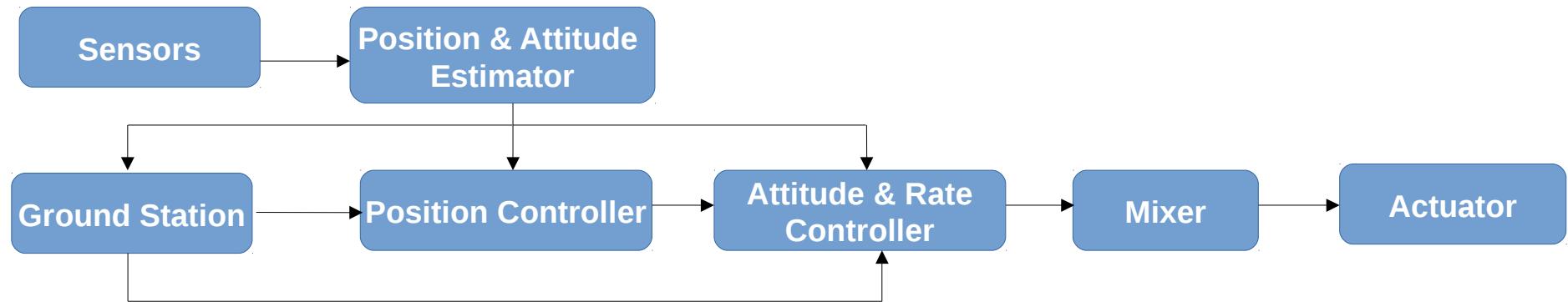
### Cons

- Need more sensors for a good measure
- Need a GPS
- Need to be wind resistant

# Problem



# General Scheme



# Quadcopter model

## Classic model

$$\ddot{x} = -\sin(\theta) \frac{1}{m} u$$

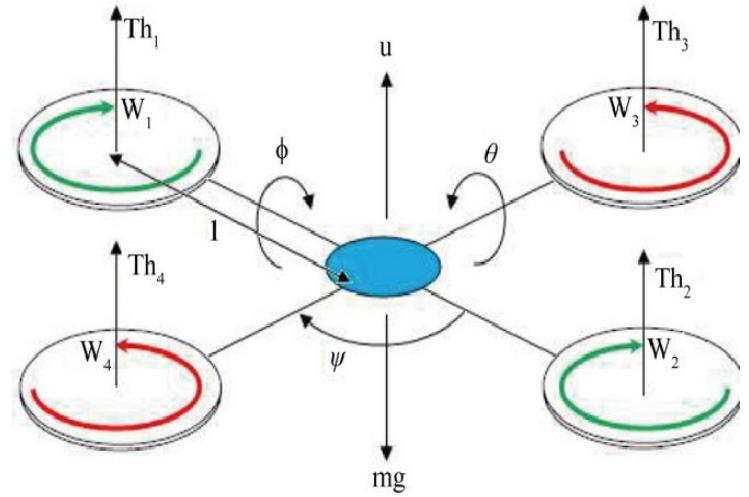
$$\ddot{y} = \cos(\theta) \sin(\phi) \frac{1}{m} u$$

$$\ddot{z} = \cos(\theta) \cos(\phi) \frac{1}{m} u - g$$

$$\ddot{\phi} = \dot{\theta}\dot{\psi} \left( \frac{I_y - I_x}{I_x} \right) - \frac{I_r}{I_x} \dot{\theta}\Omega + \frac{l}{I_x} \tau_\phi$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi} \left( \frac{I_z - I_x}{I_y} \right) - \frac{I_r}{I_y} \dot{\phi}\Omega + \frac{l}{I_y} \tau_\theta$$

$$\ddot{\psi} = \dot{\theta}\dot{\phi} \left( \frac{I_x - I_y}{I_z} \right) + \frac{l}{I_z} \tau_\psi$$



Based on quaternion

$$m\ddot{\mathbf{r}} = \mathbb{R}\mathbf{F} + \mathbf{F}_g$$

$$\dot{\mathbf{q}} = \mathbf{q} \circ \boldsymbol{\omega}$$

$$\mathbb{J}\ddot{\boldsymbol{\omega}} = \boldsymbol{\tau} - [\boldsymbol{\omega}]^\times \mathbb{J}\boldsymbol{\omega}$$

# Control law

## Nonlinear

$$\begin{aligned}
 u &= -\sigma_{z_1}(k_{d_z}\dot{z}) - \sigma_{z_2}(k_{p_z}(z - z_d)), \\
 \tilde{\tau}_\psi &= -\sigma_{\psi_1}(k_{d_\psi}\dot{\psi}) - \sigma_{\psi_2}(k_{p_\psi}(\psi - \psi_d)), \\
 \tilde{\tau}_\phi &= -\sigma_{\phi_1}(k_{d_\phi}\dot{\phi}) - \sigma_{\phi_2}(k_{p_\phi}(\phi - \phi_d)) - \sigma_{y_1}(k_{d_y}\dot{y}) \\
 &\quad - \sigma_{y_2}(k_{p_y}(y - y_d)), \\
 \tilde{\tau}_\theta &= -\sigma_{\theta_1}(k_{d_\theta}\dot{\theta}) - \sigma_{\theta_2}(k_{p_\theta}(\theta - \theta_d)) + \sigma_{x_1}(k_{d_x}\dot{x}) \\
 &\quad + \sigma_{x_2}(k_{p_x}(x - x_d)),
 \end{aligned}$$

## Linear

$$\begin{aligned}
 u &= (r1 + mg) \frac{1}{\cos \theta \cos \phi}, \\
 \tilde{\tau}_\psi &= -\sigma_{\psi_1}(k_{d_\psi}\dot{\psi}) - \sigma_{\psi_2}(k_{p_\psi}(\psi - \psi_d)), \\
 \tilde{\tau}_\phi &= \frac{k_1}{g}y_1 + \frac{k_2}{g}y_2 - k_3\phi_1 - k_4\phi_2 \\
 \tilde{\tau}_\theta &= -\frac{k_1}{g}x_1 - \frac{k_2}{g}x_2 - k_3\theta_1 - k_4\theta_2,
 \end{aligned}$$

## Based on quaternion

$$\begin{aligned}
 \boldsymbol{\tau} = & [\boldsymbol{\omega}]^\times \mathbb{J} \boldsymbol{\omega} + \mathbb{J} \mathbb{R}_e \{ \boldsymbol{\omega}_e \times \mathbb{R}_e^T \boldsymbol{\omega} + \dot{\boldsymbol{\omega}}_d \\
 & + (\boldsymbol{q}_e, \mathbf{r} \mathbb{K}_q + \mathbb{K}_{\omega e}) \boldsymbol{\omega}_e + (\mathbf{I} + \mathbb{K}_{iq} + \mathbb{K}_{\omega e} \mathbb{K}_q) \boldsymbol{q}_{e,v} \\
 & \mathbb{K}_q (\boldsymbol{q}_{e,v} \times \boldsymbol{\omega}_e) + \mathbb{K}_{iq} \mathbb{K}_{\omega e} \boldsymbol{\chi}_2 \}
 \end{aligned}$$

# Uncertainties and Disturbance Estimator

Consider the following system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= (\mathbf{A} + \Delta\mathbf{A}) \mathbf{x}(t) + (\mathbf{B} + \Delta\mathbf{B}) u(t) \\ &\quad + \mathbf{f}(\mathbf{x}, u, t) + \mathbf{d}(t) \\ \mathbf{y}(t) &= \mathbf{x}(t)\end{aligned}$$

Variable	Description
$\dot{\mathbf{x}}(t)$	State vector
$u(t)$	Control variable
$\mathbf{f}(\mathbf{x}, u, t)$	Unknown non-linear function
$\mathbf{d}(t)$	Unknown disturbances
$\Delta\mathbf{A}, \Delta\mathbf{B}$	Parametric uncertainties

The desired dynamics of the closed-loop system are given by

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_m \mathbf{x}_m(t) + \mathbf{B}_m r(t),$$

The UDE-based control law,  $U(s)$ , can be obtained as

$$U(s) = [\mathbf{I} - G_f \mathbf{B}^+ \mathbf{B}]^{-1} \mathbf{B}^+ [\mathbf{A}_m \mathbf{X} + \mathbf{B}_m R - \mathbf{A} \mathbf{X} (1 - G_f) - s G_f \mathbf{X}]$$

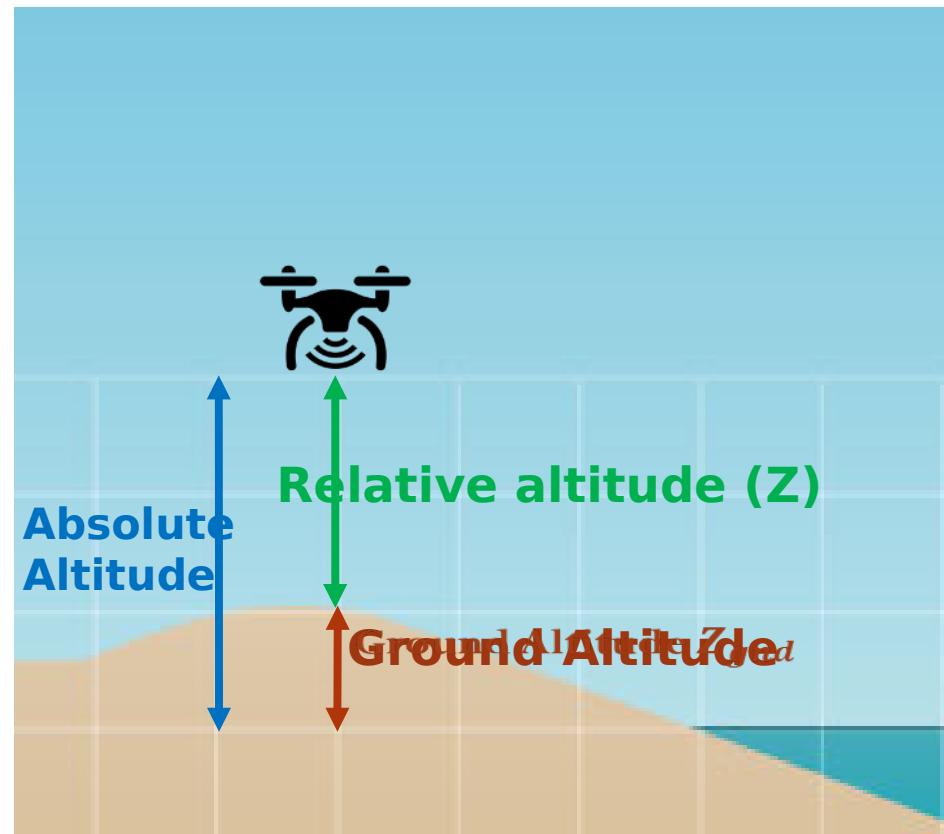
# UAV Sensing

- Problematic
- The Bebop 2
- Kalman Filter implementation
- Results



# Relative Altitude Estimation

- Autonomous Outdoor missions
- Different surfaces
- Irregular grounds
- Sensors:
  - Ultrasound
  - Barometer
- Kalman Filter



# UAV Sensing

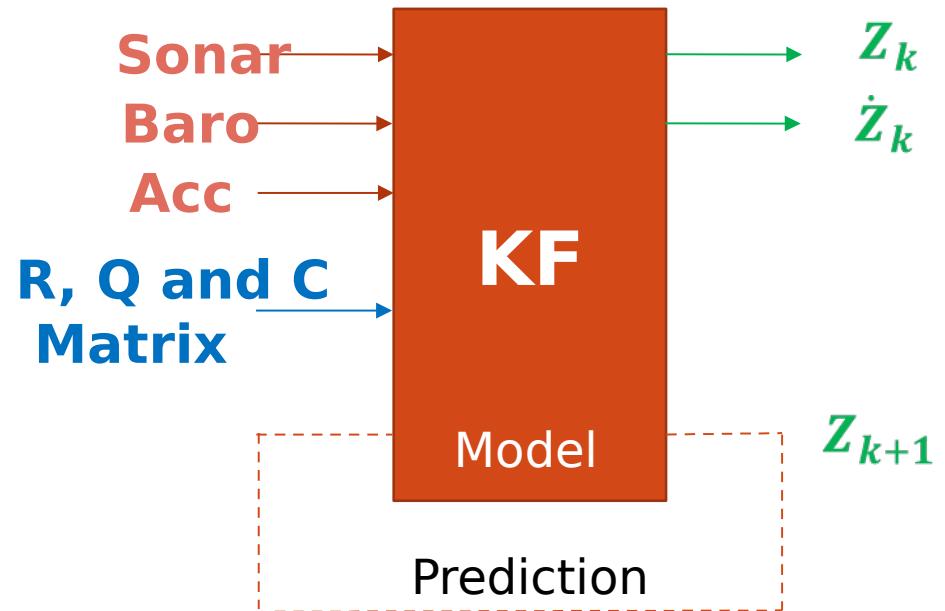
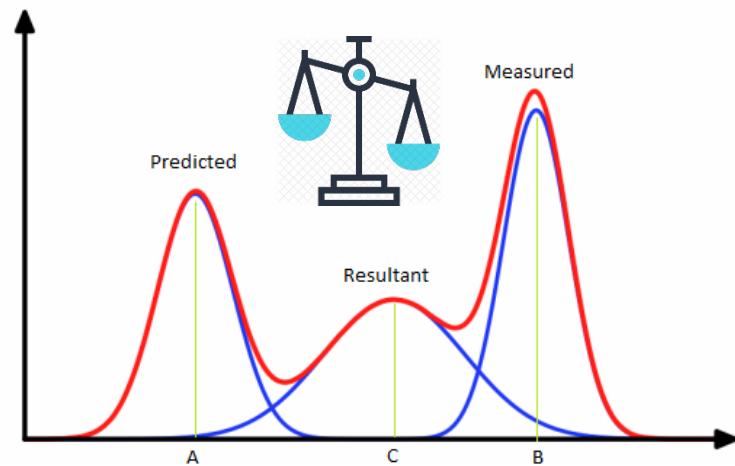


The Bebop 2:

- Made by Parrot
  - weighs 500 g
  - 25 min of autonomy
  - dual-core processor with CPU quad-core
- Ultrasound sensor
  - A pressure sensor
  - 3-axis Accelerometer
  - 3-axis gyroscope
  - 3-axis magnetometer
  - chipset (GPS + GLONASS)

# KF implementation

- The goal: Estimate a reliable relative altitude
- Sensors: Ultrasound, IMU, barometer
- The model
- KF detailed



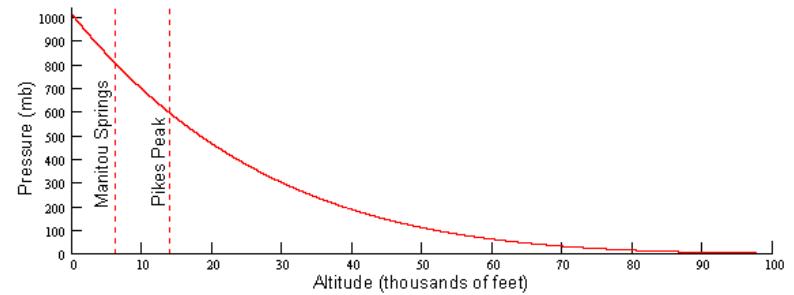
# The Sensors

- Baromet

$$P = P_b \left[ \frac{T_b}{T_b + 0.0065(h - h_b)} \right]^{5.26}$$

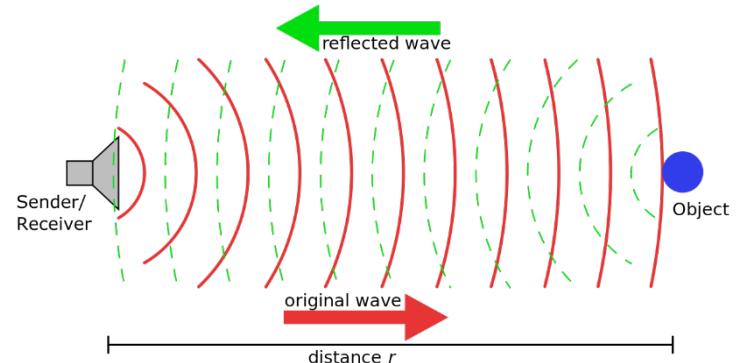
Where:

P is the current pressure  
 P<sub>b</sub> is the pressure at location b  
 T<sub>b</sub> is the temperature(K) at location b  
 h is the current altitude(m)  
 h<sub>b</sub> is the altitude(m) at location b



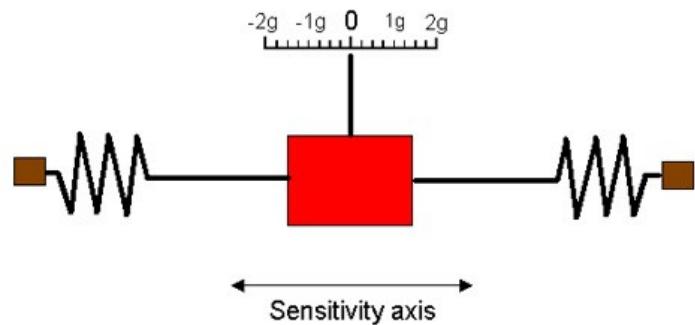
- Sonar

$$D = V_{sound} * (T * 0.5)$$



- Accelerometer

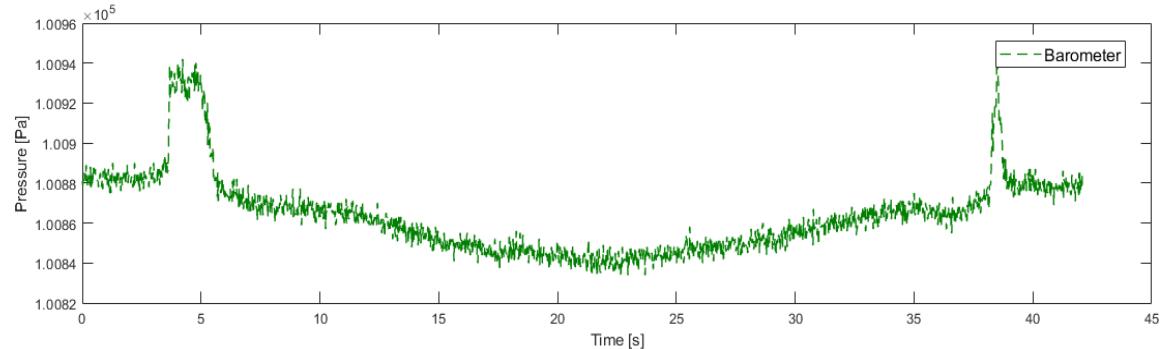
$$\ddot{Z} = Acc_z * R(\Theta, \Phi) - g$$



# The Sensors

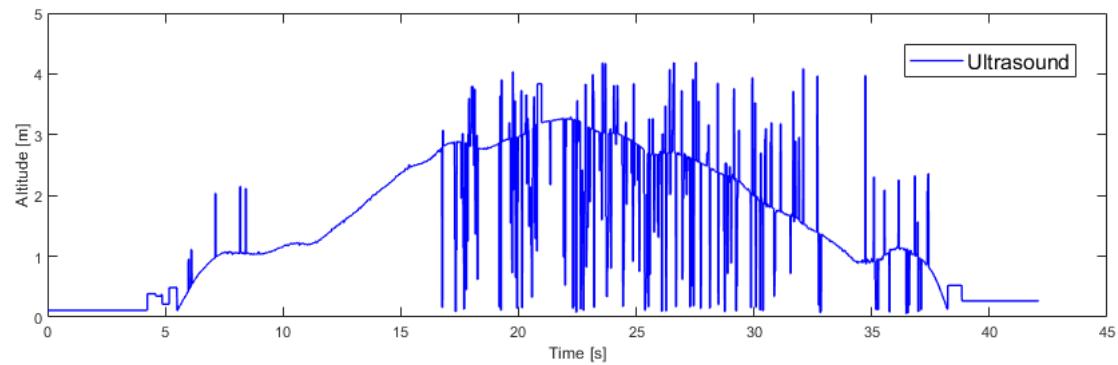
- Barometer

Pressure sensor (MS)



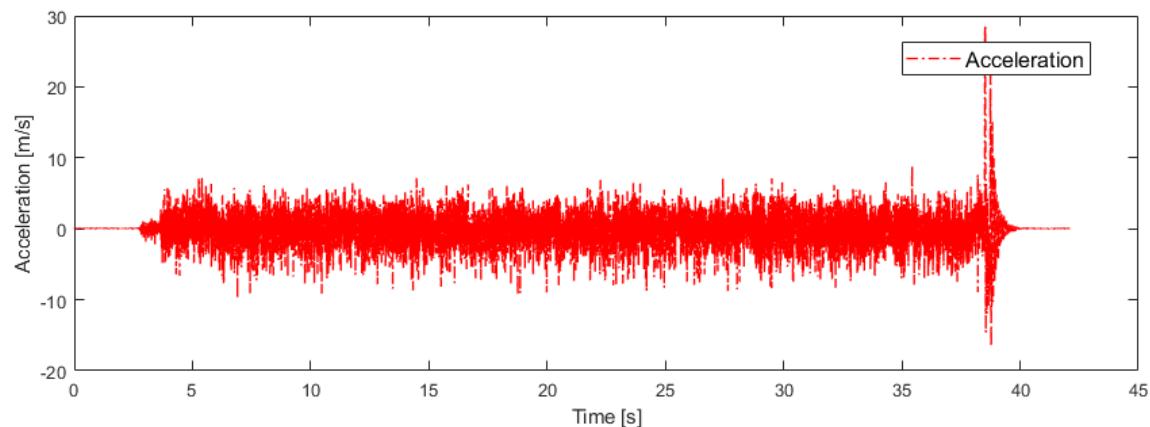
- Sonar

Ultrasound sensor



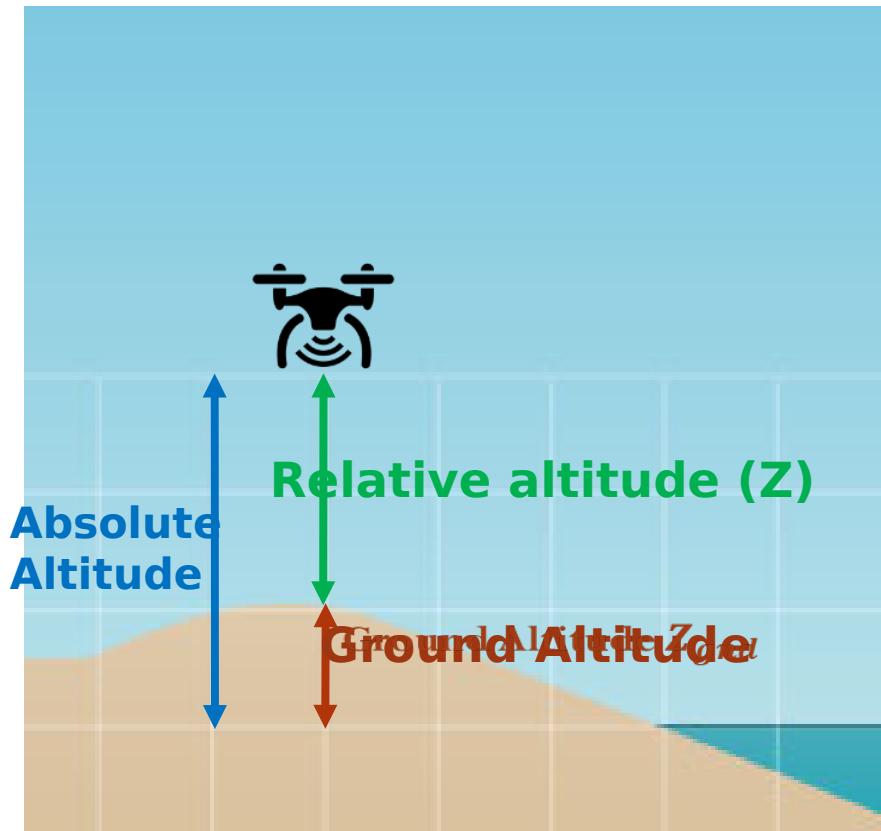
- Accelerometer

3-axes accelerometer  
(MPU 6050)



# The model

- Three states: relative altitude ( $Z_k$ ), ground altitude ( $Z_{gnd k}$ ) and altitude rate ( $\dot{Z}_k$ )



Evolution Model:

$$Z_{k+1} = Z_k + \Delta t * \dot{Z}_k + \frac{\Delta t^2}{2} \ddot{Z}_k$$

$$Z_{gnd\ k+1} = Z_{gnd\ k} + \Delta t * \dot{Z}_k + \frac{\Delta t^2}{2} \ddot{Z}_k$$

$$\dot{Z}_{k+1} = \dot{Z}_k + \Delta t * \ddot{Z}_k$$

Accelerometer

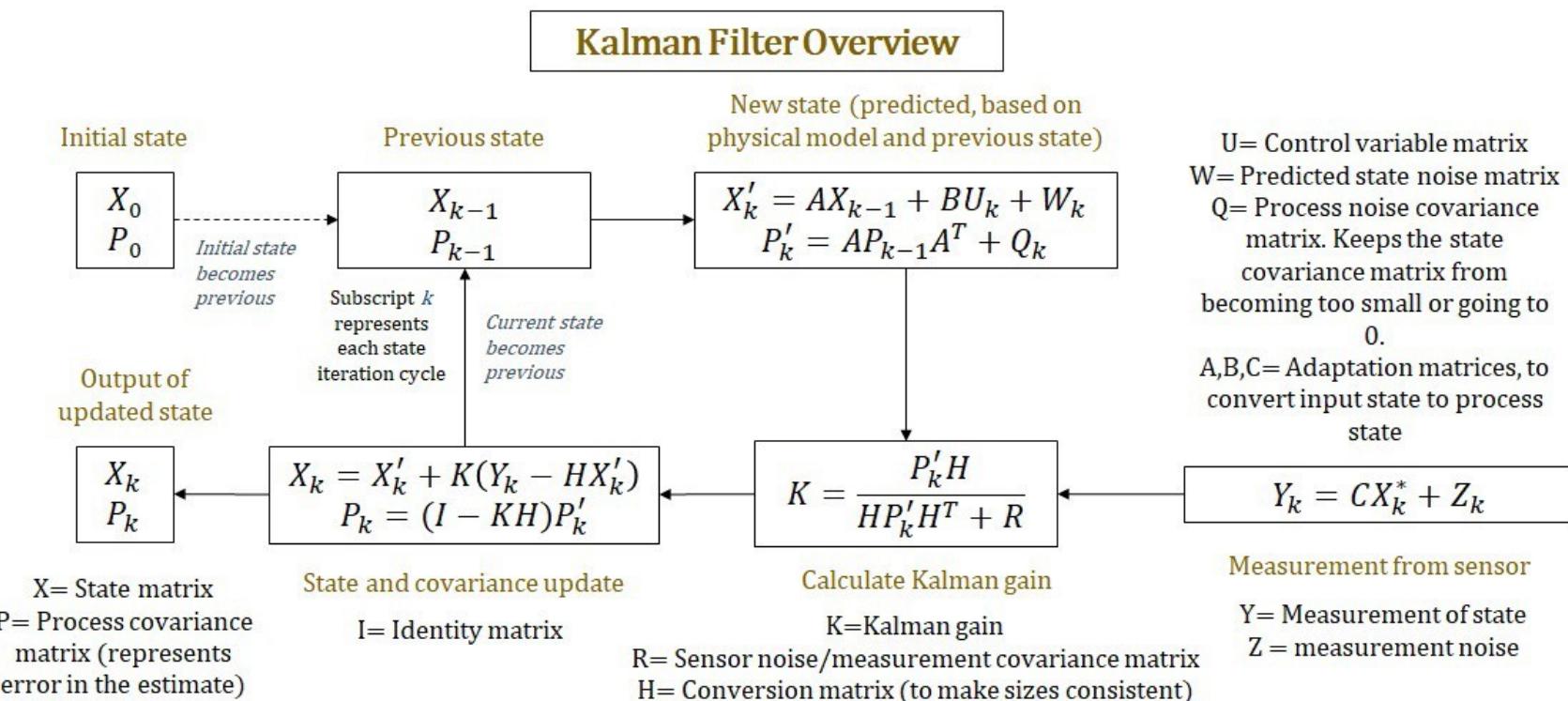
Sensor Model:

$$Alt_{son} = Z$$

$$Alt_{bar} = Z + Z_{gnd}$$

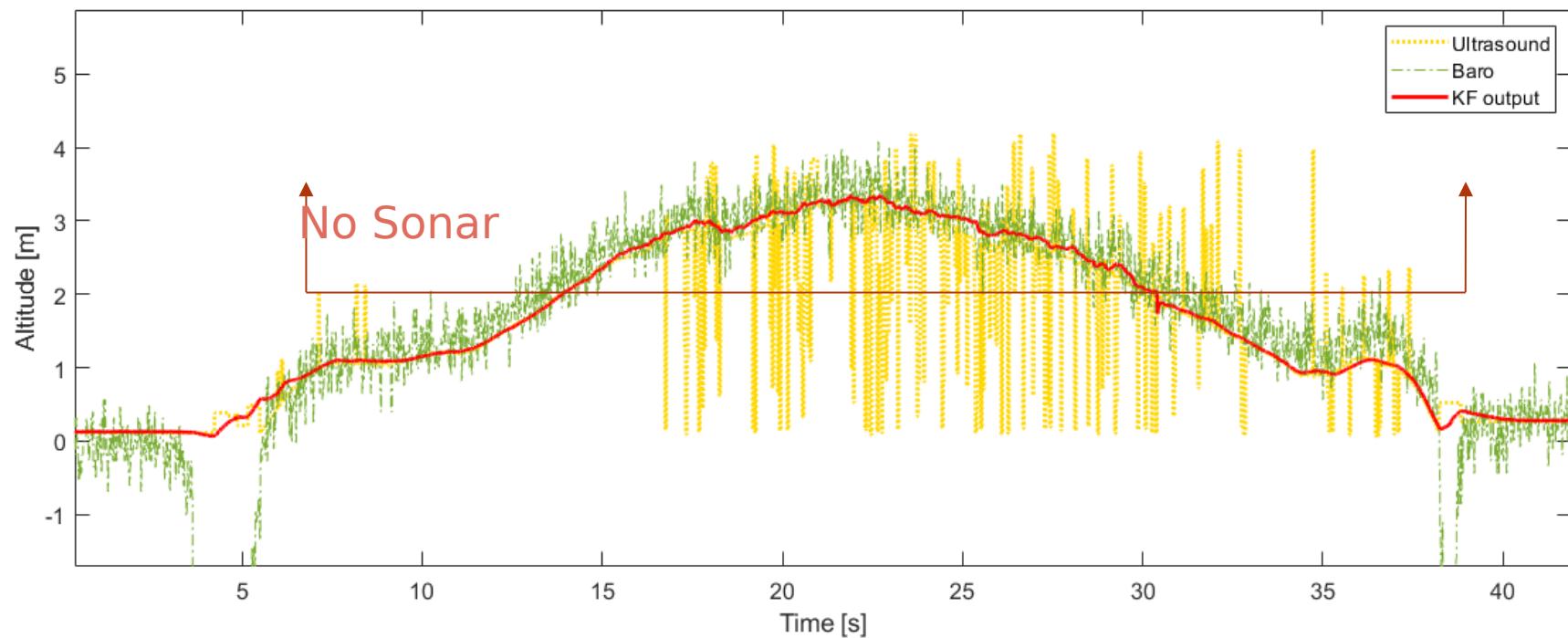
# The Kalman Filter

- Going deeper



# Results

- Flying



# Videos

**Circular Trajectory**



**KF for Altitude**



Merci  
!

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